Vortex Gas Accelerator

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The exhaust velocity of a propellant gas from an isothermal cavity can be significantly increased by a factor of approximately 1.7 above that corresponding to the usual isentropic expansion by utilizing multiple reheat by blackbody radiation from the walls during expansion. In order to reduce the necessary volume and the frictional drag with the walls during this reheat and expansion process, the gas is wrapped up in a rotational flow pattern in which angular momentum per unit mass is constant along all streamlines. As a consequence, the expansion in the nozzle corresponds to the conversion of rotational to linear momentum, rather than the usual thermal to linear. In order to absorb the radiant energy from the walls, the gas must have a sufficient optical opacity (achieved by a small contaminant addition to hydrogen) such that the gas in the cavity is approximately one radiation mean free path thick.

ROTATIONAL gas flow can be used to enhance the exhaust velocity from a constant-temperature source provided radiation-transport heating can be made greater than the vortex flow frictional decay time. In simplest terms, a partially isothermal expansion of the working fluid can be achieved as opposed to the usual adiabatic expansion. The additional enthalpy supplied to the fluid during isothermal expansion results in a higher specific impulse for rocket applications. Since this enthalpy must be added to the fluid by radiation flow, the opacity, density, frictional decay rate, and radiation intensity become the determining quantities.

There are two general forms of rotational gas flow: one is at constant angular velocity (frequently referred to as "wheel rotation") and describes the lowest order state of a gas in equilibrium inside a rotating cylinder; the second form occurs at constant angular momentum per unit mass and, to exist, requires that the rate of angular momentum supplied to the flow pattern must be large compared to the frictional drag, both with the walls and internally. The state of constant angular momentum, therefore, exists either transiently or as the result of a continuous flow of injected angular momentum and partially degraded exhaust.

In this paper we are primarily concerned with the second form of rotational gas flow in which the gas is injected tangentially at the periphery of a cylindrical cavity and expands radially toward the axis. The gas leaves the cavity by axial flow through a hole of smaller radius at one end of the cavity (Fig. 1). Both axial and radial velocities are considered as small perturbations to the primary flow, which is circular. The energy transferred in the isothermal expansion from the state of the gas at the outer wall to that at the smaller radius of exhaust is stored in kinetic energy of rotational velocity. This rotational velocity can then be converted to axial velocity in a standard nozzle (Appendix A).

It is useful to describe the density and/or pressure distribution of the wheel rotation flow in order to gain a qualitative understanding of the flow pattern near injection. In particular, one would wish to substantiate the possibility of injecting the gas tangentially at the periphery, in a thin layer, at near-constant angular momentum and pressure equilibrium with the surrounding gas. The subsequent radial and rotational flow is assumed to occur at constant angular momentum. The conditions required for this to occur will be discussed.

Imagine a hot cylindrical cavity (e.g., a reactor at temperature T) and a working gas injected tangentially at a velocity u_0. To the extent that the gas flow pattern can be approximated by a rigidly rotating body with a boundary layer in shear between the gas and the stationary cylindrical wall, then

$$dp/dr = r\Omega^2\rho \tag{1}$$

where

Ω = angular frequency
p = pressure
ρ = density

However, $p = \rho kT/m$, and letting $\rho(0)$ = density at the axis, we obtain

$$\rho(0)\ \exp(\Omega^2 r^2 m/2kT) \tag{2}$$

Defining

$$M = \left(\frac{u_0^2}{2kT/m}\right)^{1/2}$$

then

$$\rho = \rho(0)\ \exp[M^2(r^2/r_0^2 - 1)] \tag{3}$$

If we now assume for the injected gas $M^2 \gg 1$, so that the mass is concentrated in a relatively thin layer of constant angular momentum, and then ask for the distribution as this gas is heated and forced toward the axis conserving angular momentum; and if we assume sufficient radiation transport to maintain constant temperature, then the velocity distribution becomes

$$m\Omega r^2 = \text{const, or } \Omega r = u = u_0 r_0/r \tag{4}$$

Radial equilibrium from Eq. (1) demands that

$$dp/dr = r\Omega^2\rho = \rho u_0^2 r_0^2/r^3$$

or

$$\frac{d\rho}{\rho} = \frac{u_0^2}{kT/m}\frac{dr}{r^3}$$

Letting ρ_0 and p_0 be the density and pressure at the wall, then (see Fig. 2)

$$\rho = \rho_0 \exp M^2(1 - r_0^2/r^2) \tag{5a}$$

$$p = p_0 \exp M^2(1 - r_0^2/r^2) \tag{5b}$$

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In the injection nozzle, M may be large in order that the injected gas has uniform angular momentum, but after entering the cavity the gas is soon reheated by radiation. If one assumes that the injected gas expands rapidly and adiabatically in a small nozzle from a reservoir at temperature T and is then reheated "slowly" in the cavity to the same temperature T by radiation flow, then one obtains $M = [1/\gamma - 1]^{1/2}$. The gas is then assumed to leave the vortex by axial flow from the cavity at the radius r. The density distribution of Eqs. (5) implies a large expansion ratio during the transport from r_0 to r, and it is this isothermal expansion that gives rise to the enhanced gas velocity u_0r_0/r.

The rate at which energy must be supplied to the gas during this expansion is determined primarily by the mass flow, and the minimum mass flow rate in turn is determined by the viscous drag. The energy removed by drag must be smaller than the energy supplied by radiation; otherwise, the enhanced exhaust velocity cannot be achieved. The drag occurs both at the outer wall and the end walls; the internal shear flow should be laminar because the shear flow is irrotational at constant angular momentum. In addition, the density and pressure distribution of Eqs. (5) is stable against small adiabatic radial deformations. This condition is

$$-\frac{1}{\gamma}\frac{1}{p}\frac{dp}{dr} < -\frac{1}{\rho}\frac{d\rho}{dr} \tag{6}$$

which expresses the fact that, for stability, a small cell of gas transported radially outward adiabatically and in pressure equilibrium with its surrounding gas must have lower density than its surrounding gas. By inspection, the distribution of Eqs. (5) satisfies the inequality (6) provided only that the specific heat ratio $\gamma > 1$, which, indeed, is always the case. The restoring force implied by Eq. (6) is compared to the velocity shear stress in Appendix B and shown to give stability. Consequently, the internal flow should be laminar. The drag, therefore, takes place at the outer walls, and the internal flow can be considered frictionless.

The drag on the outer wall will be determined by turbulent flow and is given by

$$\tau = c_f \rho u^2 \text{ dynes/cm}^2 \tag{7}$$

where c_f is the coefficient of turbulent skin friction, which for large Reynolds number is of the order $\frac{1}{500}$.

The drag at the end wall is determined by the thickness of the Eckman layer. If this layer is less than 100 mean free paths thick (Reynolds number of 100), then the laminar shear stress becomes the drag. If the layer is thicker than 100 mean free paths, then presumably the layer will be turbulent, and Eq. (7) gives the drag. The Eckman layer is the depth of penetration of a diffusion wave (e.g., viscous shear wave) within the traversal time of sound over a distance corresponding to the local logarithmic density gradient (i.e., scale height of the exponential atmosphere). This description of the thickness of the boundary layer has been confirmed by Greenspan and Howard[1] in a linearized theory of rotational flow, but it appears physically as the result of the "buoyancy" of the nonrotating gas "rising" in the centrifugal gravitational field of the rotating gas. Since the diffusion coefficient can be expressed in terms of mean free paths and sound speed, the Eckman layer thickness becomes

$$\eth = (Dh/c)^{1/2} \tag{8}$$

where

D = diffusion coefficient $= \frac{1}{3}\lambda c$
λ = mean free path
h = scale height of density distribution $= (d\rho/\rho\, dr)^{-1}$
c = sound speed

Then

$$\eth/\lambda = [\tfrac{1}{3}(h/\lambda)]^{1/2} = \text{Reynolds number of the Eckman layer} \tag{9}$$

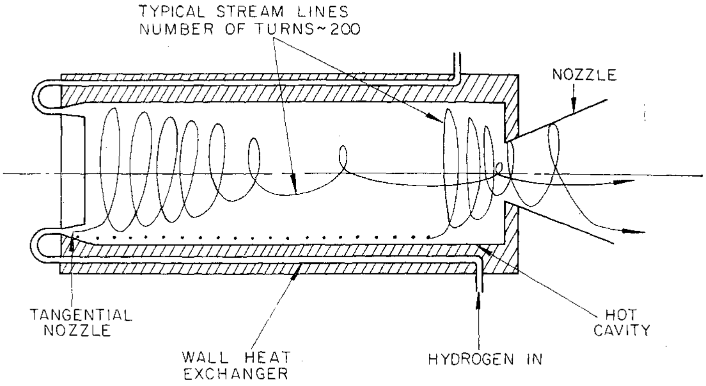

Fig. 1 Vortex gas accelerators.

and the shear stress becomes

$$\tau = \nu\rho\, u/\eth \text{ dynes/cm}^2 \text{ for } \eth/\lambda \leq 100 \tag{10}$$

where ν is the kinematic viscosity, and $\nu = \frac{1}{3}\lambda c'$($c'$ = sound speed within the layer).

If $u \geq c$, where c is measured in the unperturbed gas, then locally within the layer $c' \approx u$, and Eq. (10) becomes

$$\tau \approx (\rho u^2/3)(\eth/\lambda) \tag{11}$$

which approaches Eq. (7) for $\eth/\lambda \approx 100$.

The radiation-flow heating required to insure an isothermal angular-momentum-conserving expansion in the presence of these frictional losses can be divided into two parts, namely, 1) the heat flow required for an isothermal expansion for a mass flow rate determined by the outer wall angular momentum loss and 2) heat flow required for the end wall loss.

The end wall drag, assuming an Eckman layer thickness $\eth/\lambda \geq 100$, becomes

$$\tau_{\text{end}} = \int_{r_0}^{r} c_f \rho u^2 2\pi r\, dr = 2\pi\, c_f \rho_0 u_0^2 r_0^2 \int_{r_0}^{r} \left(\frac{1}{r}\right) \exp\left[M^2\left(1 - \frac{r_0^2}{r^2}\right)\right] dr \tag{12}$$

For $(Mr_0/r)^2 \gg 1$,

$$\tau_{\text{end}} \approx (\pi/2) r_0^2 c_f \rho_0 u_0^2/M^2 \tag{13}$$

The total cylindrical outer wall drag for a length L becomes

$$\tau_{\text{out}} = 2\pi r_0 L c_f \rho_0 u_0^2 \tag{14}$$

so that, provided the cylinder is long compared to the radius ($L \gg r_0/4M^2$), the end wall drag can be neglected. Then the radial mass flow rate per unit length must give rise to an angular momentum flow greater than the outer wall drag.

If the minimum radial mass flow per unit length is given as

$$\beta = 2\pi\, r\, \dot{r}\, \rho \text{ g/cm-sec} \qquad \dot{r} = dr/dt \tag{15}$$

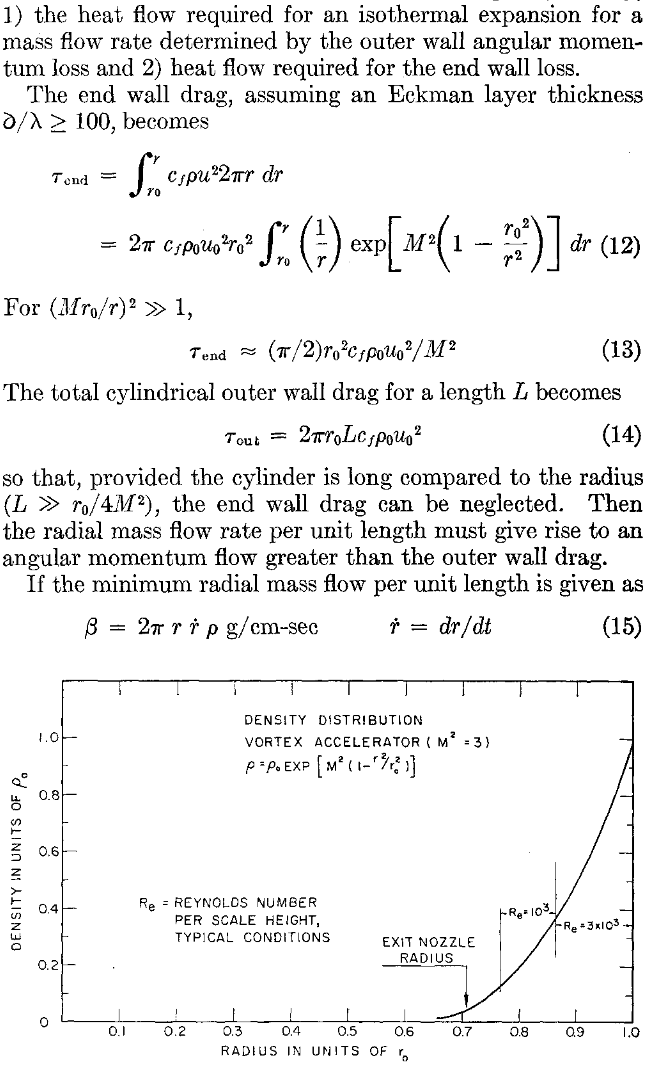

Fig. 2 Density distribution for the vortex accelerator.

Then equating drag to momentum flow at $r = r_0$ gives

$$2\pi r_0 \, c_f \rho_0 u_0^2 = 2\pi \dot{r} \, \rho_0 u_0 r_0 \quad (16)$$

and at $r = r_0$, $\dot{r} = c_f u_0$; but by Eq. (15), β is a constant, and so

$$\dot{r} = c_f r_0 u_0 \rho_0 / r\rho \quad (17)$$

The heating rate per gram corresponds to the pressure times volume work on the surrounding gas:

$$W(r) = \frac{d}{dt}\left[\int_{v_0}^{v} p \, dv\right] = \frac{d}{dt}\left[p_0 v_0 \ln\frac{v}{v_0}\right] = \frac{\rho_0 v_0}{\rho}\frac{d\rho}{dt} \quad (18)$$

By Eq. (5) and by observing that after reheat the kinetic energy of injection must be equal to the internal energy, then

$$W(r) = \dot{r}\,\frac{u_0^2}{2}(\gamma - 1)\frac{2r_0^2M^2}{r^3}\exp\left[M^2\left(1 - \frac{r_0^2}{r^2}\right)\right]$$

$$= c_f u_0^3 \frac{r_0^3}{r^4}\exp\left[M^2\left(\frac{r_0^2}{r^2} - 1\right)\right] \text{ ergs/g-sec} \quad (19)$$

For an isothermal, blackbody cavity, the opacity (μ cm^2/g) required at temperature T is determined by

$$c_l \, aT^4\mu = W(r)$$

where

c_l = velocity of light
a = Boltzmann constant

or

$$\mu = c_f r_0^3 u_0^3 \exp\{M^2[(r_0^2/r^2) - 1]\}/c_l a T^4 r^4 \text{ cm}^2/\text{g} \quad (20)$$

Assume hydrogen in a cylindrical cavity reactor with r_0 = 20 cm, r = 14 cm, T = 2800°K, $M^2 = 3$, $u_0 = 8 \times 10^5$ cm/sec, and $c_f = 0.002$. Then

$$\mu \geqslant 3 \times 10^5 \text{ cm}^2/\text{g}$$

This corresponds to a minimum cross section of 10^{-18} cm^2/molecule of hydrogen and can be compared to an optical cross section of the halogen gases of the order 3×10^{-17} cm^2 and opaque carbon macromolecules of $\backsim 10^{-14}$ cm^2. It would therefore appear feasible to meet the opacity requirements by the addition of a small percentage impurity.

The opacity might also be achieved by a carbon suspension. A carbon particle of radius $R \ll \lambda$ has a drag coefficient D for suspension in a velocity field $\dot{r}$ of

$$D = \pi R^2 \rho c \, \dot{r} \text{ dynes} \quad (21)$$

which for suspension must be in equilibrium with the centrifugal force, giving

$$\pi R^2 \rho c \dot{r} = \tfrac{4}{3}\pi R^3 \rho_p (u^2/r) \quad (22)$$

where

ρ_p = density of particle
c = local sound speed $\approx u_0$

From Eq. (22) we have

$$R = \tfrac{3}{4}(\rho_0/\rho_p)c_f(r^2/r_0) \quad (23)$$

For the conditions

$$\rho_p = 1 \qquad \rho_0 = 1.6 \times 10^{-6} \text{ g/cm}^3$$

$$c_f = 0.002 \qquad r_0 = 20 \text{ cm} \qquad r = 14 \text{ cm}$$

then $R = 2 \times 10^{-8}$ cm, corresponding to molecules comprised of a few atoms.

Separation

The small critical size of the carbon molecule for suspension in the radial velocity suggests the partial separation of heavy element additions. The critical hydrogen density for suspension of a uranium atom becomes, from Eq. (22),

$$\rho = (M/\sigma)(r_0/r^2c_f) \quad (24)$$

For the assumptions $\sigma = 10^{-15}$ cm^2, $r_0 = 20$, $r = 14$, and $c_f = \frac{1}{500}$,

$$\rho_0 = 2 \times 10^{-5} \text{ g/cm}^3$$

This is a higher density than that required for radiative power balance at T = 2940°K and so might permit a modestly higher wall temperature by the partial return of sublimated wall material.

Axial Velocity

The axial velocity required within the cavity at the exhaust radius r must be sufficient to maintain an axial mass flow equal to the radial mass flow. Assuming an exhaust area A, the axial velocity becomes

$$2\pi r_0 \, \dot{r}\rho L = \dot{z} \, A\rho$$

Therefore,

$$\dot{z} = 2\pi c_f u_0 r_0^2 L/Ar \exp[M^2(1 - r_0^2/r^2)] \quad (25)$$

For the idealized flow, the effective area becomes

$$A = \pi[r^2 - (r - h)^2] \quad (26)$$

where h, the scale height of the density distribution, is given by

$$h = [(1/\rho)(d\rho/dr)]^{-1} = r^3/2M^2r_0^2 \quad (27)$$

However, the nonuniformity of the initial angular momentum distribution is unlikely to result in a scale height as small as is implied by Eq. (27). A more realistic lower limit to h is of the order $h \approx r/2$, giving

$$\dot{z} \approx \frac{8c_f u_0 r_0^2 L}{3r^3 \exp[M^2(1 - r_0^2/r^2)]} \quad (28)$$

The limiting axial flow occurs when $\dot{z} \approx u$. Therefore, we have

$$L = \frac{3r^2 \exp[M^2(1 - r_0^2/r_2)]}{r_0 8 c_f} \quad (29)$$

For $r_0/r = 2^{1/2}$, $c_f = \frac{1}{500}$, and $M^2 = 3$,

$$L \approx 5r_0 \quad (30)$$

Energy Flow

The total energy that has been transferred to the gas can be divided into three parts:

1) The first part is the initial enthalpy corresponding to the convective heating of the gas at rest in a heat exchanger external to the cavity. The expansion of this heated gas (RT_0 ergs/g) in the injection nozzle gives rise to the kinetic energy $u_0^2/2$ ergs/g at a lower temperature ($T_1 \ll T_0$).

2) The second part is the reheat of this gas at constant pressure p_0 from the injection nozzle exit temperature T_1 to the cavity temperature T_0. This corresponds to an internal energy $R(T_0 - T_1)$ ergs/g in addition to a work term

$$\int_{v_1}^{v_0} p_0 dv = (\gamma - 1)\, R(T_0 - T_1) \quad (31)$$

resulting in a total energy increase $\gamma R(T_0 - T_1)$. This reheat of the injected gas occurs in violation of the usual gas dynamic concept that the stagnation temperature must always be less than or equal to the reservoir temperature. In this case, however, the boundary layer is thin compared to a radiation mean free path (~5%), so that, despite the higher local gas temperatures in the boundary layer, heat can still flow from the opaque wall through the hot boundary layer to the larger mass of cold gas in the freestream.

3) The third part is the work performed in the isothermal expansion as the gas flows from r_0 to r. This work is

$$p_0v_0 \ln(v_2/v_0) = (\gamma - 1)RT_0 \ln(v_2/v_0)$$

giving a final total transfer of energy to the gas of

$$\Delta W = RT_0\left\{1 + \gamma\left(1 - \frac{T_1}{T_0}\right) + (\gamma - 1)\left[M^2\left(\frac{r_0^2}{r^2} - 1\right)\right]\right\} \quad (32)$$

Considering the limitations in the L/r_0 ratio for a useful reactor, the low value of $\gamma(\approx 1.35)$ for hydrogen at a feasible wall temperature, and a reasonable injection nozzle design, the over-all energy transfer becomes $\Delta W \approx 3\ RT_0$, or an increase of specific impulse by $3^{1/2}$ above the standard nozzle.

Thrust

The thrust of such an engine is limited by the energy transfer rate from the wall. The latter, of course, is determined by the blackbody radiation from the wall for the rotational flow heating.

If T_c is the radiation temperature in the cavity, and T_w is the radiation temperature of the wall, then by Eq. (32) and assuming the reheat after injection occurs by radiation,

$$(c_l/4)\ a(T_w^4 - T_c^4) = 3\dot{r}_0\rho_0(u_0^2/2) \text{ ergs/cm}^2\text{-sec}$$

$$= 3(c_fu_0^3\rho_0)/2 \quad (33)$$

Assuming $T_w = 2940°\text{K}$, $T_c = 2500°\text{K}$, $r_0/r = 2^{1/2}$, $\gamma = 1.35$, $c_f = \frac{1}{500}$, and $M^2 = 3$, then

$$u_0 = 8 \times 10^5 \text{ cm/sec}$$

$$\rho_0 = 1.4 \times 10^{-6} \text{ g/cm}^3$$

$$p_0 = 0.14 \text{ atm}$$

The specific impulse becomes

$$\mathcal{L} = (3)^{1/2}\ u_0 \times 10^{-3} = 1400 \text{ sec} \quad (34)$$

The thrust becomes

$$T = 2\pi r_0\ L\dot{r}_0\rho_0u_0(3)^{1/2} \text{ dynes} \quad (35)$$

but, from Eq. (17), $\dot{r}_0 = c_fu_0$, so that

$$T = 2\pi Lr_0\rho_0c_fu_0^2(3)^{1/2} \text{ dynes} \quad (36)$$

For the preceding conditions

$$T = 10^5r_0^2 \text{ dynes}$$

For a mean reactor density $\bar{\rho} = \frac{1}{3}$ g/cm³, the maximum r_0 for 1-g acceleration is determined by

$$g\bar{\rho}\pi r_0^2\ L = T \quad (37)$$

or, for the preceding example,

$$r_0 = 20 \text{ cm}$$

$$L = 100 \text{ cm}$$

Therefore, a reactor built of such nested cylinders with an over-all diameter of 320 cm and 100 cm long would operate at 2.85×10^8 w with a total thrust of 4×10^3 kg.

Radiation Transfer

In the calculation of required opacity [Eq. (20)] and of the heating rate [Eq. (33)] there existed the tacit assumption that the rotating gas was less than a radiation mean free path thick. If there were many mean free paths, then the radiation diffusion equation

$$\rho Q = \bar{\nabla}\cdot\left(\frac{\lambda c}{3}\ \bar{\nabla}aT^4\right) \qquad Q = \frac{\text{energy}}{\text{g-sec}} \quad (38)$$

would have to be used, and, in general, the radiation heat flow would be degraded by a factor roughly inversely proportional to the number of mean free paths. Since this number will be shown to be of the order unity, the diffusion equation becomes a poor approximation, and the isothermal cavity radiation temperature of Eq. (33) is a better approximation.

The optical thickness of the cavity in mean free paths is

$$n = \int_{r_0}^{r} \rho\mu dr = \mu \int_{r_0}^{r} \rho\ dr \quad (39)$$

From Eq. (5), and performing the integration for $r_0/r = 2^{1/2}$ and $M^2 = 3$,

$$n = 0.12\rho_0r_0\mu \quad (40)$$

Using Eqs. (20) and (33) for μ, one obtains

$$n = \frac{0.12\rho_0r_0c_fr_0^3u_0^3 \exp[M^2(r_0^2/r^2 - 1)]}{M^2c_lar^4[T_c^4/(T_w^4 - T_c^4)](4/c_la)(3c_fu_0^3\rho_0/2)} \quad (41)$$

Letting

$$T_c^4/(T_w^4 - T_c^4) = f$$

then

$$n = \frac{0.01}{f}\left(\frac{r_0}{r}\right)^4 \exp\left[M^2\left(\frac{r_0^2}{r^2} - 1\right)\right] \quad (42)$$

For the conditions shown for Eq. (33), $f = 1$, and $r_0/r = 2^{1/2}$; then $n = 0.8$ mean free paths, confirming the isothermal cavity approximation.

In conclusion, it has been demonstrated that a significant improvement in specific impulse (1.7) can be achieved for a nuclear reactor hydrogen rocket system by utilizing a rotational flow pattern and radiation heat transfer.

Appendix A

The wall drag in the nozzle is assumed small [Eq. (7)], since the linear path of contact in the nozzle is small. As a consequence, angular momentum is conserved. Therefore, during radial expansion of the rotational flow in the nozzle, the tangential velocity $u = u_0(r_0/r)$ is reduced. Conservation of energy demands that this appear as energy of axial flow (exhaust velocity). The axial acceleration is the resultant force from the radial centrifugal stress acting on the inclined nozzle wall.

Appendix B

Turbulence

The condition [Eq. (6)] for the stability of a gravitationa atmosphere is necessary and sufficient for wheel rotation flow, i.e., no velocity shear. However, the question that arises is at what value of γ will the velocity shear implied by Eq. (4) be stabilized by the restoring force of Eq. (6)? A linear analysis certainly results in a necessary condition, but because of the damping of viscosity, the largest scale turbulence determines the stability condition. The largest meaningful radial perturbation, turble size, becomes the scale height of the density distribution, because any larger turble will necessarily be degraded by the very large changes in specific volume. Using Eq. (4) for the velocity distribution, the velocity shear energy available in one scale height h becomes

$$\frac{\Delta w^2}{2} = \frac{u_0^2}{2}\left(\frac{r_0}{r} - \frac{r_0}{r - h}\right)^2 \quad (B1)$$

Using h from Eq. (27), the energy in shear becomes

$$\frac{\Delta w^2}{2} = \frac{u_0^2}{8M^2} \frac{(r/Mr_0)^2}{[1 - 1/2(r/Mr_0)^2]^2} \quad \text{(B2)}$$

On the other hand, the energy associated with the adiabatic restoring force of Eq. (6) is

$$\Delta w/w = \Delta T/T = \gamma - 1 \quad \text{(B3)}$$

where w is the internal energy, and Δw is the change in w for e-fold expansion. However, from the injection conditions,

$$w = u_0^2/2$$

so that

$$\frac{(\Delta u)^2/2}{w} = \frac{(r/Mr_0)^2}{4M^2(\gamma - 1)[1 - 1/2(r/Mr_0)^2]^2} \quad \text{(B4)}$$

For $M^2 = 3$,

$$\frac{(\Delta u)^2/2}{w} = 0.11$$

so that the energy available in velocity shear is small compared to the adiabatic work of perturbation. As a consequence, turbulence should be suppressed everywhere. Equation (B4) is essentially the Richardson[2] number and predicts turbulence close to the value unity.

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Stability of Circumferentially Corrugated Sandwich Cylinders under Combined Loads

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Theoretical buckling coefficients are obtained for the general instability of simply-supported, corrugated-core, circular sandwich cylinders under combined loads with the corrugations oriented in the circumferential direction. The differential equations of equilibrium which are used to obtain the buckling equations are derived from the small deflection equations of Stein and Mayers which include the effect of deformation due to transverse shear. Approximate solutions to the differential equations are obtained by Galerkin's method. The resulting Galerkin equations are solved for the critical buckling coefficients with the aid of a digital computer. Curves that predict the critical buckling load are presented for axial compression, external lateral pressure, and torsion. In addition, curves are given for the combined loads of axial compression and external lateral pressure, torsion and internal or external lateral pressure, and axial compression and torsion.

Nomenclature

a, b, c, d = Fourier coefficients
A_c = cross-sectional area of core in the xz plane per inch of width, in.
A = $2t$, cross-sectional area of the facing sheets per inch of width, in.
D_c = E_cI_c, flexural rigidity per inch of width of the core in the direction of the corrugations, in.-lb
D = $Eth^2/2(1 - \mu^2)$, flexural stiffness per inch of width of equal thickness isotropic facing sheets about the centroidal axis of the sandwich, in.-lb
D_x, D_y = beam flexural stiffnesses per inch of width of orthotropic plate in axial and circumferential directions, respectively, in.-lb
D_{xy} = twisting stiffness per inch of width and inch of length of orthotropic plate in xy plane, in.-lb
E_c = Young's modulus of elasticity for core material, psi
E = Young's modulus of elasticity for facing material, psi
E_x, E_y = extensional stiffnesses of orthotropic plate in axial and circumferential directions, respectively, lb/in.
G_c = core shear modulus in plane perpendicular to corrugations, psi
G_{xy} = shear stiffness of orthotropic sandwich in xy plane, lb/in.
h = distance between middle surfaces of facing sheets, in.
I_c = moment of inertia per inch of width of the corrugations about the neutral axis of the sandwich composite, in.[3]
J = UL^2/π^2D sandwich cylinder stiffness parameter
K = buckling coefficient
L = length of cylinder, in.
m, s = integers, number of half-waves in the axial direction
n = integer, number of half-waves in the circumferential direction
N_c = force in the axial direction per inch of width acting on the middle plane of the sandwich, lb/in.
N_p = force in the circumferential direction per inch of width acting on the middle plane of the sandwich, lb/in.
N_s = shear force per inch of width acting in the middle plane of the sandwich, lb/in.
p = pressure acting on the cylinder in a directional normal to the plane of the sandwich, psi
Q_x = intensity of transverse force per inch of width acting on cross sections parallel to yz plane, lb/in.
r = radius of cylinder, in.

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